Further Thoughts on Simplicity and Complexity in Population Projection Models*

Stanley K. Smith
Bureau of Economic and Business Research
221 Matherly Hall
University of Florida
Gainesville, FL 32611-7145
Tel: 352-392-0171
Fax: 352-392-4739

* An edited version of this paper was published in the *International Journal of Forecasting*, vol. 13, pp. 557 – 565 (1997).

ABSTRACT

This article is a review of–and response to–a special issue of *Mathematical Population Studies* that focused on the relative performance of simpler vs. more complex population projection models. I do not attempt to summarize or comment on each of the articles in the special issue, but rather present an additional perspective on several points: definitions of simplicity and complexity, empirical evidence regarding population forecast accuracy, the costs and benefits of disaggregation, the potential benefits of combining forecasts, criteria for evaluating projection models, and issues of economic efficiency in the production of population projections. I believe that further discussion of these and related topics will deepen our understanding of the projection process and make population projections more useful for planning and analysis.

KEY WORDS: Demography; Population forecasting; Forecast errors; Comparative methods; Combining forecasts; Evaluation of errors.

BIOGRAPHICAL SKETCH

Stanley K. Smith is Professor of Economics and Director of the Bureau of Economic and Business Research at the University of Florida. He and his staff produce the official state and local population estimates and projections for the State of Florida. He has published articles on population forecasting in *Demography*, *Journal of the American Statistical Association*, *International Journal of Forecasting*, *Journal of Economic and Social Measurement*, and *The Review of Regional Studies*. His research interests include the methodology and analysis of population estimates and projections, the components of population growth, and the demography of Florida. He has a B.A. from Goshen College and a Ph.D. from the University of Michigan.

1. Introduction

Recently, a special issue of *Mathematical Population Studies* (Volume 5, Number 3, 1995) focused on a topic that is generally given far too little attention in the demographic literature, namely, the forecast accuracy of population projections. The theme was stated in the preface: Do complex causal forecasting models outperform complex extrapolative forecasting models? Do simple extrapolative forecasting models outperform them both?

The special issue contains six well-written and thought-provoking articles providing a variety of perspectives on population forecasting (Ahlburg 1995; Lee, Carter, and Tuljapurkar 1995; Long 1995; McNown, Rogers, and Little 1995; Rogers 1995; Sanderson 1995). The authors considered not only the forecast accuracy of simple vs. more complex models and causal vs. noncausal models, but also issues such as the costs and benefits of disaggregation in projection models, alternative methods for projecting age-specific rates in cohort-component models, choosing among alternative measures of accuracy, the development and usefulness of confidence intervals, and the importance of other criteria besides forecast accuracy for evaluating population projections.

I will not attempt to summarize or evaluate each of these articles in the present review, but will limit my comments to a few of the topics that were discussed: definitions of simplicity and complexity, evidence regarding forecast accuracy, the costs and benefits of disaggregation, and the potential benefits of combining projections. I will close with a brief discussion of why I believe this is an important topic.

Demographers often distinguish between the terms “projection” and “forecast”. A projection is typically defined as the numerical outcome of a set of techniques and assumptions regarding future trends, whereas a forecast is the specific projection the analyst believes is most likely to provide an accurate prediction of future population change. The authors of the special issue use the terms interchangeably, in essence viewing projections as forecasts. I will do the same in the present discussion.

2. Defining simplicity and complexity

Discussions of simplicity and complexity have used a variety of descriptive terms, often poorly defined: simple, complicated, naive, sophisticated, subtle, crude, elegant, and so forth. To define the terms of his discussion, Rogers (1995) followed a typology that categorized population projection models according to their mathematical and causal structures (Smith and Sincich 1992). Mathematical structures can be classified as simple or complex and models can be classified as causal or noncausal (i.e. extrapolative). Although no standard definition distinguishing simple from complex mathematical structure has been developed, there seems to be some consensus in practice; for example, linear and exponential extrapolations are generally classified as simple, whereas cohort-component and ARIMA time series models are generally classified as complex. Causal models are those in which demographic variables are affected by economic and/or other variables and noncausal models are those in which demographic variables are affected solely by their own historical values. This two-by-two matrix yields four categories of projections: simple causal, simple noncausal, complex causal, and complex noncausal. Most of the authors in the special issue followed this typology.

Long (1995) took a different approach, focusing on three types of complexity: model specification, degree of disaggregation, and the selection of assumptions and alternative scenarios. Model complexity is affected not only by the mathematical structure of the model itself, but also by the number of factors the model takes into account and the ease with which the model can be explained to data users. Degree of disaggregation refers to the extent of demographic detail provided by the projections (e.g. age, sex, race). Selection complexity is determined by the manner in which assumptions are made and the number of alternative scenarios provided. In cohort-component models, for example, assumptions regarding fertility, mortality, and migration rates may be based on their most recent values, on time series analyses of historical rates, or on theoretical models which provide predictions of future rates. The number of scenarios may vary widely; the Census Bureau has provided as many as thirty and as few as three alternative scenarios in recent sets of population projections (U. S. Bureau of the Census 1989, 1992).

Other indicators of simplicity and complexity could be considered as well, such as linear vs. nonlinear models or the degree of interaction among variables (Armstrong 1985). It is unlikely that a standard classification scheme can be developed that will cover all the possibilities. Furthermore, as pointed out by Rogers (1995), the simple vs. complex classification is really a continuum rather than a dichotomy. The issues can best be understood in relative rather than absolute terms, or "simpler vs. more complex" rather than "simple vs. complex".

That is the approach I take in the present discussion. Following the criteria mentioned above, models are classified as relatively simple or complex according to their mathematical structures, number of variables, and level of disaggregation. They are classified as causal or noncausal according to whether they are affected by other variables or only by their own historical values. Under this approach, a given model could be classified as relatively complex when compared to one model and as relatively simple when compared to another.

3. Forecast accuracy

The major focus of the special issue was the forecast accuracy of simpler vs. more complex models of population projection. However, it may be helpful to rephrase the question posed by Rogers (1995) in the title to his introductory essay: Do simple models outperform complex models? Few demographers have made this claim. Rather, the question most frequently debated is whether relatively simple models are *any less accurate* than more complex models. This is an important distinction because if simpler models are no less accurate than more complex models, they become a reasonable alternative (or even the optimal choice) for some purposes. It is not necessary that simpler models be *more* accurate than more complex models to be useful.

3.1 Projections of total population

Beaumont and Isserman (1987) argued that there is insufficient empirical evidence to draw general conclusions regarding the relative forecast accuracy of simpler vs. more complex methods for projecting total population. The authors of the special issue *seem* to share this viewpoint, although Ahlburg is the only one to say so explicitly. The other authors did not state specific conclusions on this point and, in fact, did not review much of the empirical evidence on the forecast accuracy of simpler vs. more complex population projection methods.

I disagree; I believe there is sufficient evidence to conclude that–to date–more complex models have been no more accurate than simpler models in forecasting changes in total population. There is less evidence regarding causal vs. noncausal models, but what little evidence there is suggests that they too have been about equal in terms of forecast accuracy. Not every possible combination of technique, time period, and geographic area has been evaluated, of course, but quite a few have been. What does the evidence show?

The cohort-component method is the method most commonly used by demographers for making population projections. Table 1 shows all the studies I am aware of in which the *ex post* forecast accuracy of cohort-component projections is compared with the accuracy of projections from other methods. In five of these studies (Keyfitz 1981; Leach 1981; Long 1995; Stoto 1983; White 1954) the cohort-component method is compared with one or more relatively simple extrapolation techniques and represents a relatively complex approach. In one study (Murdock, Leistritz, Hamm, Hwang, and Parpia 1984) the cohort-component method is compared with an economic-demographic projection model and represents a noncausal model. In two studies (Kale, Voss, Palit, and Krebs 1981; Smith and Sincich 1992) the cohort-component method is compared both with several other extrapolation techniques and with causal models.

White (1954) compared the accuracy of cohort-survival projections for states with the accuracy of projections from four relatively simple extrapolation techniques (linear, geometric, ratio, and apportionment). Using ten- and twenty-year horizons for 1940 and 1950, she found that errors from the relatively simple techniques were sometimes larger and sometimes smaller than errors from the cohort-survival method. These differences were generally quite small and she concluded that "no one method is clearly superior to all other methods" (p. 484).

Stoto (1983) compared five- and ten-year cohort-component projections made by the United Nations in the 1950s and 1960s for 24 regions of the world with simple geometric extrapolations. He found the simple technique to be almost unbiased and to produce errors that were equal to or smaller than those found for the more complex cohort-component projections. He concluded that "for some purposes, the simplest projection method is better than the more complicated models" (p. 18).

Leach (1981) compiled several sets of logistic curve projections and a number of component and cohort-component projections for the population of Great Britain. Comparing these projections with census enumerations for various years between 1931 and 1971, he found no evidence that the component and cohort-component projections were consistently more accurate than the logistic extrapolations. In fact, he concluded just the opposite: "the logistic curve can provide more reliable projections of total population than the component method" (p. 94). This study, however, was based on a fairly small number of empirical observations.

Using projection horizons ranging from five to twenty years, Long (1995) compared several sets of national and state-level cohort-component projections produced by the U.S. Bureau of the Census with those generated by simple geometric extrapolations. He found no consistent differences in forecast accuracy. For the national projections, the geometric extrapolations actually had smaller errors than the cohort-component projections in a large majority of the comparisons. He concluded that "a case for complexity in demographic projections cannot be made on the basis of accuracy alone" (p. 215).

Kale, et al. (1981) focused on states, evaluating the forecast accuracy of early component projections, several sets of cohort-component projections produced by the U.S. Bureau of the Census, projections from a ratio technique, ARIMA time series projections, and the economic-based projections produced by the U. S. Bureau of Economic Analysis and the

National Planning Association. Analyzing projections made between the 1930s and 1970s, with horizons ranging from five to twenty-five years, they found errors from different methods to fall within a fairly narrow range for any given length of projection horizon; projections from relatively complex or causal models were no more accurate than projections from simpler or noncausal models. They concluded that “the particular methodology giving rise to the projection appears not to matter much” (p. 12).

Murdock et al. (1984) compared population projections from 1970 to 1980 for counties in North Dakota and Texas. Two sets of projections were made, one using an economic-demographic causal model and one using a noncausal cohort-component model. They found the level of accuracy for these two sets of projections to be “nearly identical” (p. 393).

Smith and Sincich (1992) conducted the most comprehensive evaluation of population forecast errors to date. They evaluated five different sets of state projections, with launch years ranging from the mid-1950s to the early 1980s and projection horizons extending from five to twenty years. Their analysis included four relatively simple extrapolation techniques (linear, exponential, shift-share, and share of growth), an ARIMA time series model, the Census Bureau’s cohort-component model, and two economic-based causal models (Bureau of Economic Analysis, National Planning Association). They used several different measures of accuracy and bias, and conducted formal statistical tests of differences in errors by technique. They found differences in errors to be small and statistically insignificant for almost every possible combination of technique, launch year, and projection horizon. They concluded that there was “no evidence that complex and/or sophisticated techniques produce more accurate or less biased forecasts than simple, naive techniques” (p. 495).

I know of only one study finding more complex models to produce more accurate population projections than simpler models. Keyfitz (1981) compared the cohort-component projections for countries published by the United Nations in the late 1950s with projections based on the exponential extrapolation of national growth rates between 1950 and 1955. He found the U.N. projections to have smaller forecast errors than the exponential extrapolations. When he used projected growth rates for 1955-1960 instead of actual growth rates for 1950-1955 as the base for the exponential extrapolations, however, much of the difference in errors between the exponential and cohort-component projections was wiped out. It also should be noted that for long-range projections, a five-year base period has been found to produce larger forecast errors than either ten- or twenty-year base periods, especially for the exponential technique and for rapidly growing areas (Smith and Sincich 1990).

There is a substantial body of evidence, then, supporting the conclusion that more complex models generally do not lead to more accurate forecasts of total population than can be achieved with simpler models. This evidence has been drawn from studies covering a number of different projection techniques, launch years, forecast horizons, and geographic regions. Similar results regarding simplicity and complexity have been found in studies of forecast accuracy in other fields (e.g., Armstrong 1978; Mahmoud 1984; Makridakis and Hibon 1979). Fewer studies have compared causal with noncausal models of population forecasting, but they too have found no consistent differences in forecast accuracy. To my knowledge, only one study (Keyfitz 1981) has found a simpler technique to produce less accurate forecasts than a more complex technique; this study covered only one simple technique, one time period, one set of geographic areas, and used a relatively short base period for the projections from the simple technique.

I believe the weight of the evidence is sufficient to conclude that–to date–neither the sophistication of causal models nor the complexity of time series and cohort-component models has led to consistently greater accuracy in forecasting total population than can be achieved with relatively simple extrapolation techniques. There is certainly no strong empirical evidence suggesting that the opposite is true. Given the number of studies providing this evidence and the wide variety of methods, time periods, and geographic areas on which the evidence is based, I am puzzled that the authors of the special issue did not come to the same conclusion.

I do not mean to imply that all relatively simple techniques perform equally well under all circumstances, however. There are circumstances in which a *particular* simple technique may tend to produce less accurate forecasts than other simple techniques or more complex techniques. For example, exponential extrapolations have been found to have particularly large errors and a strong upward bias for places that grew rapidly during the base period (Smith 1987). The length of the base period also has an impact on the forecast accuracy of long-range projections: errors tend to be larger for projections stemming from very short base periods than for projections stemming from longer base periods (Smith and Sincich 1990). "Simple" should not be confused with "simplistic"; informed judgment is needed to determine when and how simple techniques can best be applied.

One further caveat should be mentioned. Most of the empirical studies mentioned above focused on projection horizons of five, ten, fifteen, or twenty years. Little empirical evidence exists for very short horizons (i.e. less than five years) or very long horizons (i.e. greater than twenty years). The conclusions stated here regarding the relative forecast accuracy of simpler vs. more complex techniques therefore refer only to projections of five to twenty years. Of course, population projections for horizons longer than twenty years are of dubious predictive value anyway, especially for small areas (e.g. Keyfitz 1981; Smith and Shahidullah 1995).

3.2 Projections of age groups

Although he found no consistent differences in errors by technique for projections of total population, Long (1995) concluded that cohort-component models are more accurate than simpler models for projecting population by age group. This may yet prove to be true, but I believe further testing is required. Long based his conclusion on a comparison of errors for two age groups (15-19, 60-64) from two projection models: the Census Bureau's cohort-component model and an exponential model based on the growth rate of each age group in the year immediately prior to the launch year. Not surprisingly, he found errors to be much larger for the exponential model than for the cohort-component model.

I believe a better simple model can be developed by using a ten-year rather than a one-year base period and by extrapolating by cohort rather than by age group. Under this approach (Hamilton and Perry 1962) cohort growth rates are calculated by dividing the population age i in year t by the population age i-10 in year t-10. These rates can then be applied to each age group in year t to provide projections by age in year t+10. The population less than age ten can be projected in a number of ways (e.g. applying the ratio of children less than age 10 to women age 15-44 in year t). As a final step, the projections by age (or by age and sex) can be adjusted to add to the projections of total population produced by some other technique(s), thereby reducing the large errors and upward bias sometimes found in projections derived from growth-rate extrapolation models (e.g. Smith 1987).

This is still a relatively simple model. It requires only data by age (or age and sex) in two consecutive censuses rather than age-specific fertility, mortality, and migration rates and assumptions regarding future changes in those rates. In spite of its simplicity, it incorporates the effects of population momentum and provides projections of the demographic characteristics of a population, making up for two of the major shortcomings of the relatively simple techniques often used to project total population. Lee et al. (1995) and McNown et al. (1995) discuss simpler vs. more complex applications of cohort-component models; the Hamilton-Perry approach is simpler yet. A comparison of age group projections from the Hamilton-Perry approach with those from other cohort-component models would provide an interesting and potentially useful test of simpler vs. more complex models.

4. Disaggregation

Demographic disaggregation refers to the breakdown of population stocks and flows into their component parts. It often plays a central role in population projection models (e.g. age-specific birth, death, and migration rates in cohort-component models). Although disaggregation is essential for evaluating individual components of population growth, Lee et al. (1995) questioned whether the details of disaggregation are genuinely helpful for forecasting total population or mainly a distraction from what might be more important issues, such as long term historical trends, foreseeable structural changes, and potential environmental constraints to future growth. They suggest that complex disaggregations may actually conceal dynamic regularities that otherwise would be apparent (p. 220).

I share the concerns expressed by Lee et al., especially as they relate to projections for states and local areas. Migration is by far the most volatile component of population growth for states and local areas and is the major determinant of differences in their growth rates. The evidence cited in the previous section strongly suggests that complex breakdowns of past migration data do not make up for the uncertainty inherent to predicting the direction and size of future changes in migration rates. In addition, it should be noted that the data required by highly disaggregated models are frequently unavailable for subcounty areas (e.g. census tracts).

This issue has particular relevance for small areas (e.g. counties and subcounty areas). If improvements in the accuracy of small-area population projections are to be achieved, they will most likely come not from higher levels of disaggregation, but rather from incorporating into the projection process factors such as expected structural changes, potential constraints to growth, and recent growth trends in contiguous areas. Perhaps a set of “leading demographic indicators” based on these factors can be developed to guide the projection process. Although increased complexity has not led to any consistent improvements in forecast accuracy to date, complexity based on factors such as these may lead to future improvements in the accuracy of small-area population projections.

5. Combining projections

Ahlburg (1995) called for more research on the benefits of combining projections from different models to create population forecasts. I agree. Combining can be done in many ways, using different models or techniques, different specifications of the same model or technique, simple averages vs. weighted averages, historical weightings vs. subjective weightings, and so forth. A number of studies have concluded that combining forecasts often leads to greater accuracy and less variability than can be achieved by individual techniques alone (e.g. Armstrong 1985; Mahmoud 1984; Makridakis and Winkler 1983; Voss and Kale 1985).

One approach to combining that may be particularly useful for population projections is the "composite" method suggested by Isserman (1977). This method is based on the assumption that some models or techniques perform substantially better (or worse) than others under particular circumstances or for places with particular characteristics. For example, exponential extrapolations tend to have an upward bias and relatively large errors for rapidly growing areas, especially when projections cover a long time horizon (Smith 1987). If consistent patterns can be observed frequently enough to draw general conclusions, forecasts for particular places can be based solely on the models or techniques expected to be most accurate for those types of places rather than using the same models or techniques for all places. One study found this "composite" approach to produce more accurate forecasts for census tracts than an average based on the same techniques for all places (Smith and Shahidullah 1995). Research in this area may lead to further improvements in population forecast accuracy, especially for small areas.

6. Why it matters

Why does the question of simplicity vs. complexity matter? I believe it is important for two reasons, one related to scientific verification and the other related to the efficient use of resources. For many years there has been a common perception among both the producers and consumers of population projections that more complex models are generally more accurate than simpler models (e.g., Birch 1977; Irwin 1977; Pittenger 1980; Keyfitz 1981; Beaumont and Isserman 1987). Given the evidence discussed above, I do not believe this perception is valid. The question of forecast accuracy, however, is one that can be answered empirically–at least for past time periods. Although issues such as choosing appropriate measures of error, adjusting for degree of difficulty, and including representative projection techniques must be resolved, these issues are no more difficult than those confronted in most scientific studies. By conducting rigorous tests of various population projection models and techniques, the level of scientific knowledge regarding forecast accuracy can be raised. This is an important objective in and of itself.

The answer to the simplicity-complexity question also has important implications for the use of scarce resources. As pointed out by Long (1995), Rogers (1995), Ahlburg (1995), and others, there are many criteria besides forecast accuracy upon which population projection models can be judged, such as fairness, timeliness, use of recent data, provision of sufficient detail, consideration of relevant variables, usefulness for policy-making, consistency with other types of projections, reasonableness of assumptions, internal consistency, cost of development, ease of explanation, and suitability in providing a base for other projections. These are all valid criteria. There are undoubtedly purposes for which relatively simple projection models do not provide an acceptable alternative to more complex models.

For many purposes, however, the most important factor is simply the expected degree of forecast accuracy (e.g. Yokum and Armstrong 1995). If relatively simple models or techniques can provide forecasts that fulfil the needs of the data user and are no less accurate than more complex models or techniques, they not only provide a viable alternative but may be the optimal choice because they are typically much less expensive in terms of input data and production time. For example, compiling and cleaning up age-specific mortality, fertility, and migration data for small areas is a tedious and time-consuming process, whereas collecting total population data for several points in time is not. In such instances simple models or techniques represent a more efficient use of scarce resources than more complex models or techniques. This benefit will be particularly important when projections for many geographic areas have to be made (e.g. county projections made by a state demographic agency; census tract projections made by a market research company).

One downside risk of "simplicity" should be mentioned. The use of complex models and techniques may make the analyst appear to be intelligent and well-informed, whereas the use of simple models and techniques may make him/her appear to be stupid, lazy, or poorly trained. Complex models and techniques also provide an imposing array of details behind which to hide when things go wrong, whereas simple models and techniques are transparent and may leave the analyst open to charges of overlooking important factors that could have improved the forecasts. Analysts who use relatively simple models or techniques not only must have the expertise to know when they are appropriate and how best to use them, but sometimes may need a thick skin as well.

7. Conclusion

Recent research–including the articles in this special issue of *Mathematical Population Studies*–has made important contributions to the literature on the production and evaluation of population projections. Many questions remain unanswered, however. Why is it that complex models and techniques generally have been no more accurate than simpler models and techniques in forecasting total population? What are the circumstances in which some models or techniques *are* generally more accurate than others? Can this information be incorporated directly into the projection process? What are the circumstances in which the benefits of complex or causal models make them worth their high costs? How can the uncertainty inherent to population forecasts best be measured and this information conveyed to data users? Answers to these and similar questions will improve our understanding of population growth and demographic change, improve the quality of population projections, and enhance the usefulness of those projections to data users.

Table 1. Forecast accuracy of projections of total population: a summary of conclusions from empirical studies

Conclusion	Studies
1) Complex models are *no more* accurate than simpler models.	Kale, et al. (1981); Leach (1981); Long (1995); Smith and Sincich (1992); Stoto (1983); White (1954).
2) Complex models are *more* accurate than simpler models.	Keyfitz (1981).
3) Causal models are *no more* accurate than noncausal models.	Kale, et al. (1981); Murdock et al. (1984); Smith and Sincich (1992).
4) Causal models are *more* accurate than noncausal models.	None.

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