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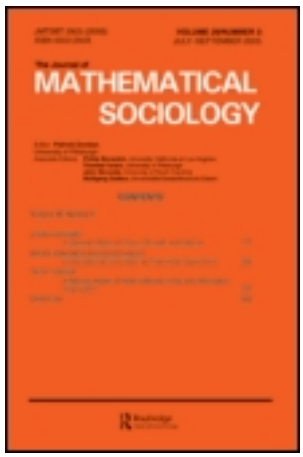

The Journal of Mathematical Sociology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmas20

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Published online: 26 Aug 2010.

To cite this article: François Lorrain & Harrison C. White (1971) Structural equivalence of individuals in social networks, The Journal of Mathematical Sociology, 1:1, 49-80, DOI: 10.1080/0022250X.1971.9989788

To link to this article: http://dx.doi.org/10.1080/0022250X.1971.9989788

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Journal of Mathematical Sociology
1971, Vol. 1, pp 49–80

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Printed in Birkenhead, England

STRUCTURAL EQUIVALENCE OF INDIVIDUALS IN SOCIAL NETWORKS†

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The aim of this paper is to understand the interrelations among relations within concrete social groups. Social structure is sought, not ideal types, although the latter are relevant to interrelations among relations. From a detailed social network, patterns of global relations can be extracted, within which classes of equivalently positioned individuals are delineated. The global patterns are derived algebraically through a 'functorial' mapping of the original pattern. Such a mapping (essentially a generalized homomorphism) allows systematically for concatenation of effects through the network. The notion of functorial mapping is of central importance in the 'theory of categories,' a branch of modern algebra with numerous applications to algebra, topology, logic. The paper contains analyses of two social networks, exemplifying this approach.

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By interrelations among relations is meant the way in which relations among the members of a social system occur in characteristic bundles and how these bundles of relations interlock and determine one another.‡ By understanding is meant distilling simpler patterns at a higher level of abstraction—simpler not only in having fewer constituents but also in exhibiting interrelations which are more regular or transparent.

Practicable ways of carrying out analyses with data have been developed, in the form of computer programs (Heil 1970). However this approach is not just a novel technique of data reduction; rather, as will presently be explained, it follows from our concern with a set of sociological problems.

Our treatment, which stems from a tradition of algebraic analysis of kinship systems (Weil 1949, White 1963, Courrège 1965, Boyd 1969), differs from other

†This work was supported under grants GS–448 and GS–2689 to H. C. White from the National Science Foundation, which is gratefully acknowledged. Special thanks are due to Scott Boorman, whose unfailing criticism gave us much cause for reflection and led us to numerous revisions. Discussions, in seminars or otherwise, particularly with Daniel Bertaux, Mark Granovetter, and John MacDougall, have also been helpful. However, the undersigned are clearly responsible for any imperfections that may remain. This paper is the product of the synthesis of an unpublished paper by H. C. White ('Notes on Finding Models of Structural Equivalence', 1969) and certain results taken from an unpublished paper by F. Lorrain ('Tools for the Formal Study of Networks, I.', 1968) and from the latter's doctoral dissertation (Lorrain 1970, in press). The determinant stimulus that led us to the ideas set forth in the present paper came from Boyd's 1966 dissertation (the core of which was subsequently published—Boyd 1969), where the decisive step was made, introducing and exemplifying a particular type of reduction of a network. Friedell's (1967) notion of 'office structures' within organizations also pointed in the same direction.

‡For a brilliant formulation of the problem, see Nadel 1957. However he emphasizes mainly cultural interrelations of role sets.

approaches to the analysis of networks—such as graph theory, the theory of electric circuits, or production scheduling—in that it handles the points of view of all nodes simultaneously. It also differs from other approaches in the study of social networks in that it consists of concurrent dual treatments of individuals on the one hand and relations on the other. Although the elements of our framework are stable expectations held by persons, it should be possible to deal in this way with macroscopic flows either of material resources or of abstractions—such as information, uncertainty, attitudes—as seen by an observer. Such flows involve aggregates of individuals in equivalent positions within the abstract global structure. Moreover these aggregates will vary according to the particular abstract point of view taken; many such overlapping global patterns can coexist or conflict at the same time.

In order to do justice at least in part to the bewildering complexity of social structure, it is important to take into account the possibly very long and devious chains of effects propagating within concrete social systems through links of various kinds. It is partly owing to these indirect effects, together with the largely extrinsic driving force of membership renewal†, that social structures and processes can so vastly transcend the individual consciousness of actors and investigators. Another consequence is that cultural systems of cognitive orientations, or of values, or of norms, however complex and however crucial, can only constitute a small part of the social phenomenon. Although some global structural aspects may be culturally recognized and be expressed in strong and important social norms, this is not necessarily so for all such aspects. It is exceedingly important to realize that, as soon as an abstract cultural framework is inscribed within a set of concrete persons, coming and going, being born and dying, wholly unanticipated consequences may result. Thus when we speak of the global network structure of a social system, we have in mind the overall objective logic of this system as it exists concretely in a population of so many individuals related in such and such ways. This is why simultaneous treatment of individuals and of relations is essential. One of the deepest misunderstandings in prevailing sociological theories is their failure to distinguish effectively between individuals and social positions.

RELATIONS AND GRAPHS

The total role of an individual in a social system has often been described as consisting of sets of relations of various types linking this person as ego to sets of others (see for example Gross et al. 1958). Let us represent individuals as nodes which are not distinguished by intrinsic attributes such as sex; this restriction will be discussed later. Let us draw an arrow with a label from one person to another to represent a type of role relation. Thus total roles will appear as sets of variously labelled arrows, not as higher order structures. 'Counterpart' role relations directed back to the given person are simply treated as parts of the sets of role relations of the others. In other

†White (1970) deals extensively with membership renewal in organizations, particularly emphasizing the duality of individuals and social positions, and showing that objective flows of vacancies through an organization are the crucial phenomenon, not careers.

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words, total roles are broken down into directed ties of different kinds occuring within pairs of persons.

An important requirement is that these links correspond to clear expectations among the population considered. Definition of different types of role relation is a question of substantive theory related to the particular purposes of a study (for an example of a systematic analysis see Davis 1969). Deciding which particular ordered pairs of nodes exhibit a given type of tie is an empirical question, but one based on abstraction, since ties of several different types may hold at the same time between a given pair of nodes. The problem of measurement of social relations is a difficult one; unfortunately most existing sociometric tests seem hardly acceptable as tools for the measurement of systematic structure.†

The pattern of occurences of a relation of a given type within a network, i.e. the set of all ordered pairs of nodes exhibiting this type of relation, constitutes a directed graph. Hereafter the term *graph* will mean directed graph and will be used interchangeably with the term *binary relation*, which simply means a set of ordered pairs of nodes. Let us assign to each type of role relation a capital letter as a label. A specific tie in the graph of a relation labelled, say, R will be represented in the usual way as aRb, where a, b are nodes and the direction of the tie is by convention from a to b.

Definition of social relations will be tied to the given population by *one major restriction*: if the graphs for two types of social relation are identical, i.e. if they consist of exactly the same ordered pairs of nodes, the two will be treated as a single type.

Of course it is conceivable that two culturally distinct role relations may happen at some time to have identical graphs in a given population. However it does not seem very probable that they would remain distinct for a long time, if the coincidence of their graphs persists. (Compare this with Gause's ecological axiom, Slobodkin 1961, p. 123.) This is a sociological argument. Roles are not isolated abstract entities: they exist only in so far as they have dynamic reality in a concrete population. A role involves more than just two persons: the concrete patterning of flows along the graph of a relation as well as the ways these flows concretely interlock and overlap with other types of flow are probably more important in shaping the content of a role relation than even the most articulate cultural specifications. The above restriction is not a matter of emphasis on extension rather than on intension of role concepts; the point is rather that, as far as internal flows of a system and aggregates of individuals are concerned, it is not necessary to distinguish in the abstract what is nowhere concretely distinguished. However such a point of view requires an important precaution. The domain of people and relationships considered must be relatively bounded and complete: it makes but little sense, in this perspective, to select an arbitrary subset of a social system and to submit it to the type of analysis we propose, because the structure of global flows could thereby be critically distorted. Naturally this in no way excludes that the given domain be also part of a wider system and interact or even overlap with other systems.

By definition a role relation must imply reciprocal expectations. (See Nadel 1957,

†As clearly shown by P. W. Holland and S. Leinhardt in an unpublished paper, 'Masking: The Structural Implications of Measurement Error in Sociometry,' 1969.

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Chapter 2, for an unusually clear discussion of role as a concept.) Formally, if a role relation labelled R is recognized then there necessarily exists a *converse* role relation, which can be labelled R^{-1}, such that, for any nodes a, b, aRb if and only if $bR^{-1}a$. Of course, by definition, $(R^{-1})^{-1}$ is precisely the relation R. This *principle of reciprocity* is another manifestation of the reality, of the objectivity sui generis of social phenomena. However this principle may not fully apply to 'relations' corresponding to relatively imprecise 'sociometric choices'; such a case will be presented and analyzed in a later section.

The converse relation R^{-1} must be distinguished from the 'counterpart' roles mentioned earlier, which are simply role relations of the same generic kind as R. Several distinct role relations and their respective converses may simultaneously hold on certain pairs: e.g. aRb, $aS^{-1}b$, aTb, together with $bR^{-1}a$, bSa, $bT^{-1}a$.

Symmetry is a particularly important aspect of social relations. Consider a type of role relation R. Suppose that expectations of one person of a pair in the graph of R toward the other are the same as the other's toward the first, and let this be true for every pair of nodes included in the graph of R. Then not only is each tie symmetric, but the graph as a whole is symmetric, i.e., for any nodes a, b, aRb if and only if bRa. Thus the graph of the converse R^{-1} coincides with the graph of R: $bR^{-1}a$ if and only if bRa. The major restriction just stated then applies: since R and R^{-1} coincide, they are treated as a single inclusive role—call it R still. In this sense, a symmetric role relation has no reciprocal role relation. Receiving and acknowledging the friendly expectations projected to one by a friend, for example, is treated as an intrinsic part of one's own projection of the same friendly expectations to him, so long as all pairs in the population are symmetric in this way.

Merger of R and R^{-1} when R is symmetric can be used as a prototype in arguing again for the one major restriction. The great complexities and subtleties of a role relation, between two particular persons or as viewed in general in a culture, are being excluded as far as possible from this analysis. Our approach is orthogonal to Friedell's (1969), who is concerned with the deep structure of mutual perceptions in social groups. The whole thrust is toward how patterns of role relations fit together in a population. From this point of view, it is not important to distinguish passive from active on one person's side of a role tie unless the two aspects occur separately somewhere in the population.

This argument suggests an ambiguity which must be resolved when defining a role relation. Let F be the graph of role ties found in a population using a definition of the role relation of friend elicited from informants and their culture. In many pairs, no doubt, there will be a symmetric tie with the role relation holding in both directions. Let this subset of pairs and their ties be represented by a subgraph of F called S. A substantive judgment must then be made as to whether the S ties are so different from the others that there are really two distinct role relation types, even though they are not recognized explicitly by the population. Davis and Leinhardt† have adopted this strategy of splitting S from F (see also Davis 1968). A typical example of this type of situation is the distinction between reciprocal use of the second person singular of

†J. A. Davis and S. Leinhardt, 'The Structure of Positive Interpersonal Relations in Small Groups', unpubl. paper, 1968.

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verbs in verbal interaction and use of the second person singular reciprocated by use of the second person plural or third person, a distinction of outstanding importance in many languages. Once again the point is that the nature and content of ties is closely related to the pattern of their distribution among individuals, to which close attention must be given in the course of analysis as well as in field-investigation.

COMPOUND RELATIONS IN SOCIAL NETWORKS

Dynamics and structure in a population cannot be captured from the mere count of role relations by type—as proposed by Wolfe and Schenk (1970), for example—any more than from a census of attitudes or attributes of individuals. The problem is how to get at the interweaving of pair relations into the complex tapestry of social structure and process. The theory of electric circuits comes to mind as an analogue, particularly in the elegant formulation of Kron (1939), ably simplified and restated by LeCorbeiller (1950), or in the formulation of Slepian (1968). There are three crucial defects in the analogy, exploration of which can guide further treatment of social ties.

1. Ties in social networks often generate other ties or the elimination of other ties, unlike branches in an electric circuit which are the given skeleton of a network changed only by an outside agency, the designer.

2. The driving forces in an electric network are external to its logic, scattered arbitrarily in branches of it, whereas the activation of the population rests on impulses generated at each of the nodes, whether or not with external stimulation.

3. The nature of a tie, as argued earlier, depends in part on the pattern of all ties of that type among the population in comparison with the patterns of all ties of various other types. More generally, the nature of the ties between a given pair of persons depends on their (and others') perceptions of how these ties fit in with other role relations among the population. Whereas the nature of a branch in an electric network is fixed by design, although of course its operation depends on what flows develop elsewhere (perhaps directly through mutual inductance) and breakdown can occur because of overload.

Composition of Relations and the Algebraic Notion of Category

Everyone recognizes the reality of indirect ties, ties to one's boss' friend, or one's roommate's relative, or one's ally's enemy. Indirect ties are even sometimes themselves institutionalized and part of a role system, as in the case of kinship ties. The relation of such indirect or secondary ties with one another and with the direct or primary ties is an obvious way to capture the interweaving of role relations into a structure on the population.

Denote RS the secondary relation linking a to b, implied by the existence of a node x such that aRx and xSb. RS will be referred to as the *compound* of R and S, and this operation of compounding will be called *composition*. Whenever aRx and xSb then necessarily $a(RS)b$; however the converse of this will often not be true: in

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general, if $a(RS)b$ then there does not necessarily exist an x such that aRx and xSb: for example, the compound relation RS might happen to be quite strongly institutionalized in its own right, so that if x left the system in one way or another a would keep his RS tie to b. This is important and relates to our argument that a relation involves more than just two persons (or three, for that matter). We shall later encounter a number of examples of compounds of relations having this property.

Let us then extend our conception of composition of ties so that there may be compounds of any order, such as RSS, $RRR^{-1}S$, etc.; refer to this extended operation simply as the *composition operation* of the network considered. By the major restriction stated previously, there can be only a finite number of such compound types of relation—because there is only a finite number of possible binary relations on a finite population. Certain equations then must hold among strings of relation symbols: otherwise there would be as many relation types as there are such strings, that is an infinite number. For example, if the graph of R coincides exactly with the graph of RR then by our major restriction R and RR will be treated as a single type of relation and the following equation will hold: $R = RR$. This particular equation implies that R is transitive: whenever aRb and bRc then always aRc. In addition the social nature of converse relations is such that all equations of the type $(RS)^{-1} = S^{-1}R^{-1}$ must hold in any network.

We shall deal later with the problem of defining a composition operation when initially only primary, *generator* ties are explicitly given. For the moment, let us suppose that we have a full composition operation for a given network.

This operation represents the basic logic of *concatenation*, the basic logic of *interlock* among the relations constituting the network. Of course this operation is meaningful only in so far as the generator relations are clearly defined; otherwise it would make no sense to distinguish, say, RS from RR, or from SRS^{-1}, etc. The composition operation constitutes one of the main differences between our approach and the more usual 'sociometric' treatments of social networks; it stands for quite a different level of structure.

Note that compound relations, as here defined, are independent of the particular intermediary people involved: the nature of such compound relations depends only on the types of tie that are concatenated to form the compound relation in question. This is because all nodes can be active sources of information and motivation (see point 2 above).

Note also that the compound of two relations, say, X and Y is not necessarily defined: if it is never the case that three nodes a, b, c of the given network are such that aXb and bYc, then the relation XY simply does not occur in the network, there is no point in even speaking of it, in so far as this particular network is concerned. Again, even if the compound XY would happen to make sense culturally, if for some length of time X and Y persisted in concatenating nowhere in the network, it is doubtful that the compound XY would retain any effective social reality.

Let us refer to either generator or compound types of relation as *morphisms*. Some compound morphisms will be explicitly recognized, others will not—which in no way implies that they have less real an effect. These morphisms, together with their composition operation and their graphs on the set of nodes constitute a *category*. More precisely:

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Definition. A *category* C is constituted by a class $CObj$—the elements of which may be called nodes but are usually called *objects*—, together with a class $CMor$—the elements of which are called *morphisms*—, and provided with a structure of the following type.

1. To each ordered pair (a, b) of objects is assigned a subset $(a, b)Mor$ of $CMor$. The elements of $(a, b)Mor$ are referred to as the morphisms *linking* a to b. If M is a morphism linking a to b, this may be indicated by writing aMb. It is understood that every morphism appears as a link for at least one pair of objects.

2. If objects a, b, c and morphisms M, N are such that aMb and bNc then there is amorphism MN linking a to c, called the *compound* of M and N. The compound MN depends only on M and N: it is independent of any particular objects a, b, c that might be involved in their concatenation. If on the other hand there are no objects a, b, c through which M and N concatenate as above, then the compound of the two morphisms is *undefined*, the expression 'MN' has no meaning. This operation of compounding of morphisms is called the *composition operation* of C.

3. If objects a, b, c, d and morphisms M, N, P are such that aMb, bNc, and cPd, then $(MN)P = M(NP)$, so that there is no ambiguity in speaking of the compound of M, N, and P through a, b, c, d: it is a unique morphism—which may as well be denoted simply by MNP, deleting the parentheses—, linking a to d: $a(MNP)d$. This is the property of *associativity* of the composition operation. Of course, by 2, MNP is independent of the particular a, b, c, d involved in the concatenation of the three morphisms.

The concept of category is an important one because it takes a network *as a network*, combining together in a unit the *three* levels of objects, of morphisms, and of concatenation of morphisms. In a category all objects are considered simultaneously, while graph theory considers only particular cycles and paths linking definite nodes; neither does graph theory consider any classification of paths into types according to the types of the links concatenated.†

A final remark. There is no objection to seeing composition of morphisms as developing in time (see point 1 above on electric networks). Generation of a full pattern of indirect effects—even if none of them would be of a conscious nature—can conceivably take some time; in such a case compound morphisms would become meaningful only relative to a long enough period.

†This definition of a category differs in some respects from the usual definition of a category in mathematics: according to the present definition a category does not necessarily have identity morphisms and a morphism can apply to more than one ordered pair of objects. (A standard reference on categories is Mitchell 1965). This definition renders possible a richer dialectic between objects and morphisms and is better adapted to psychological and sociological applications. Thus, at the immediate level of a social network represented as a category in the sense just defined, the mathematical theory of categories in its present form has but little relevance, although the spirit is the same. However in the study of whole classes of such categorical networks category theory becomes directly relevant and proves to be quite useful in establishing certain results (see Lorrain 1970). Hereafter the term category will refer exclusively to the notion as defined here. Although the major restriction stated in the section 'Relations and Graphs' in general does not apply to a category, it will be understood to apply to all categories hereafter considered.

Composition When No Explicit Composition Operation Is Given

Frequently, in mathematics, structures consisting of certain elements are considered, where a certain type of combination of these elements would be useful for structural analysis but no explicit rule for such combination is given. Often in such cases rules of combination are defined in a purely abstract, formal manner and are used as a basis for further treatment. We shall follow a similar course for social networks.

Let a social network be given, containing various types of role relations, but where there seems to be no natural composition operation available. First consider as many compound relations as there are 'words' that can be formed by using as letters the labels of the role relations given at the start—call these *generators*—, without any limitation to the length of words or to the order of letters within them. Consider two generators R, S. There is no alternative, in this case where only primary relations are given at the start, but to define the graph of the compound relation RS to be the unique graph such that: for all nodes a, b, $a(RS)b$ if *and only if* there exists a node x such that aRx and xSb. In other words the *graph* of RS here is exactly the composition of the *graph* of R and the *graph* of S. Similarly, define the graph of a tertiary relation RSS to be the composition of the graph of RS and the graph of S, or equivalently as the composition of the graph of R and the graph of SS, composition of graphs being associative. And so on.

Denote by G our (non-empty) set of generator relations. Once again, although an infinite number of words can be formed with elements of G as letters, only a finite number of graphs will be generated by composition of generators, since there is only a finite number of possible binary relations on a finite population. This set of graphs —denote it by S_G—, considered together with the operation of composition of these graphs, constitutes a *semigroup*: the composition of any two elements of S_G is again an element of S_G and composition is furthermore associative. The infinite set of words can be partitioned into a finite number of classes such that any two words in the same class have exactly the same graph. By the major restriction stated above, all the words within a class will be considered to represent a single type of relation, to which will correspond a unique element of S_G. This means that in general the same type of relation can be denoted by more than one word. Such partitions of sets of words are dealt with in the theory of free semigroups and their homomorphic reductions (see for example Clifford and Preston, pp. 40 ff.) and in extensions of this theory to the more general case of categories.

Now define a category C_G as follows. The objects of C_G will be the nodes of our network. With only one possible exception to be discussed below, the morphisms of C_G will be all the types of relation generated from G, which are of course in one-to-one correspondence with their graphs, the elements of S_G. A morphism M will link an object a to an object b if and only if a is tied to b in the graph of M. The composition operation of morphisms will follow exactly the composition operation of the semigroup of graphs S_G. If we add only the slight modification just mentioned and to be presently described, this structure is seen to satisfy all requirements of our definition of a category. Such a category structure captures adequately the network properties in which we are interested here and can form a useful basis for further analysis. A concrete example will be described in detail in a moment and will be analyzed further n this paper.

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One particular binary relation is important here: the empty or *zero* relation, i.e. the relation consisting of the empty subset of the set of all ordered pairs of nodes. This relation—denote it O—has the property that, for any binary relation R, $RO = OR = O$. Now it may happen that O is one of the elements of S_G, i.e. that there exist certain types of compound relation not exhibited by any pair of nodes in the given network. In such a case O must be excluded from the morphisms of C_G, if C_G is fully to satisfy point 1 of the definition of a category. This is the modification that was just announced. Hence the compound of two morphisms is zero in the unmodified C_G if and only if in the modified C_G this compound is undefined (see point 2 of the definition of a category). Hereafter C_G will denote only the modified C_G. It may seem artificial to exclude O from C_G, but we shall see later that such an exclusion is important; the fact that two types of relation never concatenate in the network is of crucial sociological significance, because it indicates a form of disjunction, break, or decoupling within the structure. Moreover, contrary to the C_G and S_G case, in general it is not sufficient to add a zero to the morphisms of a category to obtain a semigroup; we shall encounter examples of such categories below, pp. 66 and 69.

Another graph is of special importance: the *identity* relation I. This is the apparently trivial graph linking every node to itself: it consists exclusively of 'self-loops'. I has the property that for any binary relation R on the given set of nodes $RI = IR = R$; i.e. I acts as an *identity element* in semigroups of binary relations. Usually I will not be an element of S_G, however; whether or not to add I to S_G—and add it also, as a new *identity morphism*, to C_G—depends on the interpretation of the model to be constructed. Identity morphisms will play a central role in our treatment of social networks as categories.

Note, finally, that, although in C_G composition of morphisms coincides by definition with composition of their graphs, this is not in general true of all categories. This remark was already made at the beginning of the previous subsection and must once again be emphasized. Concrete examples of such categories will be considered later.

An Example

Let us compute the category C_G for a population of five persons and the set of two generators P and P^{-1} illustrated in Figure 1. Exactly twenty-one distinct types of relation are generated from P and P^{-1}: the shortest words representing them are listed in Table 1 and the graphs of a few of them are given in Figure 2. In Table 1

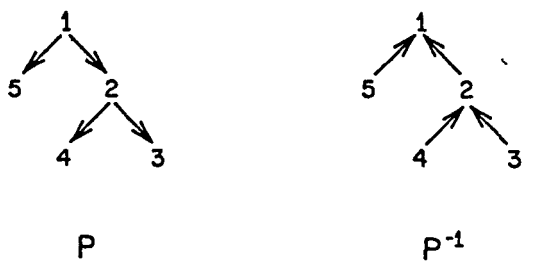

FIGURE 1 A hierarchical tree.

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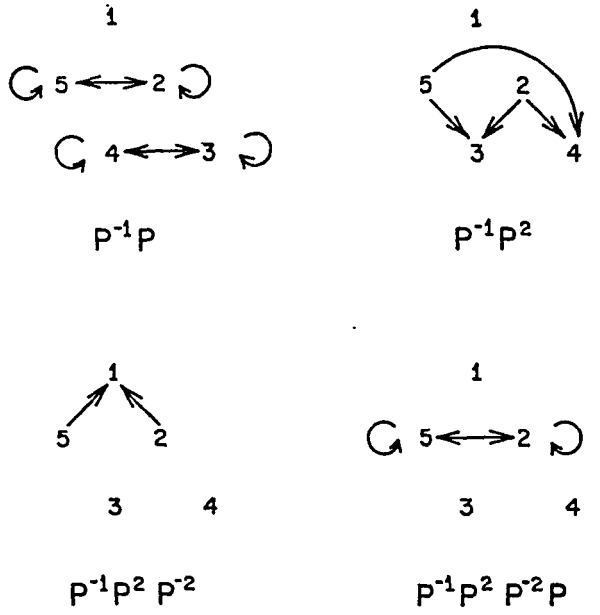

FIGURE 2 Graphs of four of the types of relation in Table 1

and Figure 2, P^2 stands of course for PP, P^{-2} for $P^{-1}P^{-1}$, etc. It is not hard to show that any other 'word' has the same graph as one of these twenty-one words and hence, by our major restriction, is considered to represent the same type of relation. Six equations are sufficient to equate any word to one of the twenty-one types: $PP^{-1}P = P$, $P^2P^{-2}P^2 = P^2$, $P^3 = O = OP = PO$, and the ones equating the converses of these: $P^{-1}PP^{-1} = P^{-1}$, $P^{-2}P^2P^{-2} = P^{-2}$, $P^{-3} = O = P^{-1}O = OP^{-1}$. An example

TABLE 1

List of the twenty-one distinct types of relation generated by P and P^{-1} of Figure 1.

$P^{-1}P$, $P^{-2}P^2$; I (the identity, adjoined to C_G, not generated)
PP^{-1}, $PP^{-2}P$, $P^{-1}P^2P^{-1}$, $P^{-1}P^2P^{-2}P$, $PP^{-2}P^2P^{-1}$
P^2P^{-2}
P, $P^{-1}P^2$, $PP^{-2}P^2$
P^2P^{-1}, $P^2P^{-2}P$
P^{-1}, $P^{-2}P$, $P^{-2}P^2P^{-1}$
PP^{-2}, $P^{-1}P^2P^{-2}$
P^2
P^{-2}
O (generated for example as P^3)

Note.—The assignment to separate lines and the further grouping by brackets are used in a later section, together with the adjoined identity I.

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of a deduction from these equations is: $PP^{-1}P^2P^{-2}P=(PP^{-1}P)(PP^{-2}P)=P(PP^{-2}P)$ $= P^2P^{-2}P$. In such deductions, there is complete freedom in the use of parentheses since composition in S_G—which here is isomorphic to composition of types of relation —is associative. The empty relation O will be excluded from the set of morphisms of C_G and the identity morphism I (linking each object to itself) will be added, so that C_G will have twenty-one morphisms. Table 2 gives the distribution of morphisms among pairs of objects in C_G. In Table 3 is shown a part of the composition table of

TABLE 2

Distribution of morphisms among ordered pairs of objects in the category C_G generated from P and P^{-1} of Figure 1.

$(1,1)Mor = \{I, PP^{-1}, P^2P^{-2}\}$
$(2,2)Mor = \{I, P^{-1}P, PP^{-1}, PP^{-2}P, P^{-1}P^2P^{-2}P, PP^{-2}P^2P^{-1}\}$
$(5,5)Mor = \{I, P^{-1}P, P^{-1}P^2P^{-2}P\}$
$(3,3)Mor = (4,4)Mor = \{I, P^{-1}P, P^{-2}P^2\}$
$(1,2)Mor = \{P, P^2P^{-1}, P^2P^{-2}P\}$
$(1,5)Mor = \{P, P^2P^{-2}P\}$
$(2,3)Mor = (2,4)Mor = \{P, P^{-1}P^2, PP^{-2}P^2\}$
$(5,3)Mor = (5,4)Mor = \{P^{-1}P^2\}$
$(1,3)Mor = (1,4)Mor = \{P^2\}$
$(2,5)Mor = \{P^{-1}P, PP^{-2}P, P^{-1}P^2P^{-2}P\}$
$(3,4)Mor = \{P^{-1}P, P^{-2}P^2\}$

Note.—The morphisms associated to an ordered pair such as (2,1) are the converses of the morphisms of the pair (1,2): $(2,1)Mor = \{P^{-1}, PP^{-2}, P^{-1}P^2P^{-2}\}$.

TABLE 3

Part of the composition table of the category C_G generated from P and P^{-1} of Figure 1.

	P	$P^{-1}P^2$	$PP^{-2}P^2$	$P^{-1}P$	$P^{-2}P^2$
P^{-1}	$P^{-1}P$	$P^{-2}P^2$	$P^{-2}P^2$	$P^{-2}P$	⊠
$P^{-2}P$	$P^{-2}P^2$	$P^{-2}P^2$	$P^{-2}P^2$	$P^{-2}P$	⊠
P^2	⊠	⊠	⊠	P^2	P^2

the morphisms of C_G: this table is the same as the composition table of S_G, except that where a cross occupies a cell in the former—meaning that composition of the two particular morphisms involved is undefined in C_G—a zero would occur in the

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table of S_G. More than 43% of the cells in the composition table of C_G are undefined: there is a large amount of decoupling among morphisms in C_G (compared to 0% decoupling in a semigroup).

Twenty-one types of relation among five people may seem an unreasonably high number. However this is still orders of magnitude smaller than the total number of possible binary relations on a set of five elements, namely $2^{5\times 5}$, i.e. more than 33 millions! Moreover, we shall use C_G only as a base from which to extract the simpler global patterns we are looking for. The advantage of using a base such as C_G is that it takes systematic account—albeit in an apparently too refined manner—of the way the generator relations concatenate in the original network. The possibility must be left open that the many compound types of relation generated—more than one of which may occur at the same time within a single pair of nodes—correspond to substantially distinct types of relation, flow, or effect. This relational multiplicity within a single pair might be consciously manipulated, or it might correspond to an objective ambiguity involving conflicts of relation quite independent of the will of individuals, etc.

The number of morphisms of C_G depends critically on the structure of the graphs of the generator relations. If in the previous example the ties of P were not oriented ties but symmetric ties, then only three distinct relation types would be generated, S_G being then the cyclic semigroup of order three and period two (Clifford and Preston, p. 19). The size and structure of C_G can also vary immensely if even only one node is deleted or added: if node 2 was deleted from the previous example, C_G would have only five morphisms: I, P, P^{-1}, PP^{-1}, $P^{-1}P$. This serves as a caution applying to the delimitation of the network to be studied; this is an important caveat, already expressed above.

Endomorphisms

At first sight, self-loops in the graph of a type of relation, such as the loop $1(P^2P^{-2})1$ in the hierarchy example, might not seem relevant. However these loops—or *endomorphisms*—certainly represent real feedback effects (whether individuals be aware of them or not) which form an integral part of the structure and can be of crucial importance in its dynamics. Furthermore, certain endomorphisms are in a sense part of an individual's consciousness of his position in the structure, they are part of his identity; endomorphisms thus acquire a great significance when searching for possible persons in the network with which a given person is most likely to identify or ally. Endomorphisms will be of considerable moment in the following.

AGGREGATION OF RELATIONS AND STRUCTURAL EQUIVALENCE AMONG INDIVIDUALS: FUNCTORS

Reductions

Our purpose will now be to derive models of aggregation of relations and of individuals, by mapping C_G—or any representation of a social network as a category—onto a smaller, simpler, *reduced* category. This will involve two simultaneous map-

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pings: one of objects and the other of morphisms.† Naturally, if the composition operations of the original and the reduced categories are to mean anything, these mappings should also be compatible with these two operations. Such a mapping of a category onto another is a *functorial reduction*, hereafter simply called *reduction*. Reductions are examples of *functors*, a more general kind of mapping between two categories. Reductions and functors will be defined more precisely below.

A reduction of a category constitutes a 'point of view', a 'cross-section', a 'projection' of this category, leaving out certain aspects of its structure and retaining others. Many such reductions, such points of view on a category are possible, more or less refined and overlapping to varying degrees. Virtually nothing can be done with a category such as C_G outside of the context of its reductions and their interrelations; only in this context can its sociological fruitfulness be assessed.

One might expect the social process to operate in such a way that the enormous cognitive and emotional complexities involved in multiple compound relations in a social network C inevitably become structured into the simpler pattern of some reduction of C, where possibly whole classes of nodes become structurally equivalent and hence may be considered as units. Certain reductions of C might perhaps delineate latent macrostructures within C, which could become realized socially under certain circumstances (examples of this will be described later). Some reductions might even happen to be invariant in time, while C could be in a state of incessant change; this would express in a rigorous way the idea of social continuity. An example of such a time-invariant reduction, to which we shall return later, is the structure of clans and clan relationships in societies with certain types of kinship system. More than one time-invariant reduction of C could exist, some of which individuals in C might be quite unaware of, or might be strongly reluctant to acknowledge.

Reduction of Morphisms

Let a category C be given, representing a social network. We shall now examine in more detail what should be meant by a reduction of the morphisms of C, leaving aside reduction of objects until the next section. Interdependence of reduction of objects and reduction of morphisms will appear fully only when sociological criteria for reduction will be discussed in the last part of this paper, where examples of reduction will be considered.

Let D be another category, with the same objects as C but with perhaps different morphisms, i.e. $CObj = DObj$ but possibly $CMor \neq DMor$. Suppose f is a mapping of $CMor$ *onto* $DMor$ (see Footnote). If f is to represent a meaningful reduction of C to D then at least the following three conditions should be satisfied.

1. Whenever aMb in C then $a(Mf)b$ in D: every link between two objects in C becomes a link in D.

Thus a reduction of C to D maps every occurrence of a morphism (type of relation) M in C to a unique type of relation Mf in D.

†A *mapping* m of a set A into a set B is any rule assigning to every element x of A a unique element of B, denoted xm and called the *image* of x. m maps A *onto* B if every element of B is the image of an element of A; in this case B obviously cannot have more elements than A has, so that in general if m is onto it can be viewed as a *reduction* of A to B.

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2. Whenever aQb in D then there exists a morphism N of C such that $Q = Nf$ and aNb: every link between two objects in D is the image of a link between these two objects in C.

Conditions 1 and 2 can also be expressed as follows: the graph of a morphism Q of D is exactly the *union* of the graphs of all the morphisms of C of which Q is the image.

Condition 1 implies that whenever the compound of two morphisms M, N of C is defined in C, then the compound of Mf and Nf is necessarily defined in D (see point 2 of the definition of a category). We shall also require a third condition of f:

3. If the compound MN of morphisms M, N of C is defined in C then the composition operations are such that $(MN)f = (Mf)(Nf)$.

This necessitates an important remark. In a semigroup, composition of any two elements is defined. A *homomorphism* of a semigroup S into a semigroup T is a mapping g of S into T such that, for all elements X, Y of S, $(XY)g = (Xg)(Yg)$. However, condition 3 does not imply that a reduction of the category C_G defines at the same time a homomorphism of S_G onto a reduced semigroup; this can be seen as follows. In condition 3 the equation $(MN)f = (Mf)(Nf)$ has no meaning if MN is undefined; now MN is undefined in C_G if and only if $MN = O$ in S_G; thus the equation $(XY)f = (Xf)(Yf)$ can be required of elements X, Y of S_G only in the case where $XY \neq O$. Such a relaxation of the homomorphism requirement for semigroups was made by Boyd (1969), who did not use categories. However this extension of the notion of semigroup homomorphism is still insufficient to cover all cases of reduction of categories: as already remarked, the set of morphisms of a category does not necessarily constitute a semigroup, even if a zero is added, and even if this category is a reduction of C_G: this will be made clear in an example, p. 69. A homomorphism of a semigroup onto another is only a particular case of reduction of morphisms in a category.

Let C be again any category, and suppose that there is a morphism E of C such that, for every morphism M of C, both EM and ME are defined and $EM = ME = M$. In other words, E acts as an identity morphism in C (although its graph can be different from that of the identity relation I and the latter is not necessarily a morphism of C). Then Ef has exactly the same properties in D: this is due to condition 3 together with the fact that f maps *CMor onto DMor*. Thus the image, through a reduction, of an identity morphism is again an identity morphism.

We shall say that two morphisms M, N in *CMor* having the same image in *DMor* —i.e. such that $Mf = Nf$—are *identified*. This will also be denoted by the equation $M \equiv N$. To identify two morphisms means to lump them together, widening their definition so that both become the same morphism in the image. When dealing with social networks, all such identifications of morphisms should occur in dual pairs: i.e. $M \equiv N$ if and only if $M^{-1} \equiv N^{-1}$. Obviously the relation $\equiv$ among the morphisms of C is an equivalence relation; the image morphisms are in one-to-one correspondence with the equivalence classes of $\equiv$. In general every such identification entails other identifications if condition 3 above is to be satisfied; in particular, if the compounds MN and $M'N'$ are defined in C and if $M \equiv M'$ and $N \equiv N'$ then by condition 3 we must also have $MN \equiv M'N'$. Conversely, it is easy to show that any

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equivalence relation on *CMor* having the latter property defines a possible reduction of *C*.

It is not always necessary or convenient to compute a reduction in detail. Often a few basic identifications are sufficient, from which all other identifications and hence all the properties of the reduction can be deduced; the basic identifications then constitute the axioms of the reduced structure. It is often not easy or even impossible to compute the reduction implied by a set of identifications; an example is the theory of free groups and their reductions (Coxeter and Moser 1965), which is applied in the theory of a type of classificatory kinship system (White 1963). However, when *C* is finite and given in detail, it is easy to compute the reduction entailed by any set of identifications; computations too long to do by hand are done on a computer (Heil 1970). The exact procedure of such a computation will be made clear when dealing with examples later on.

Structural Equivalence of Objects

Objects *a*, *b* of a category *C* are *structurally equivalent* if, for any morphism *M* and any object *x* of *C*, *aMx* if and only if *bMx*, and *xMa* if and only if *xMb*. In other words, *a* is structurally equivalent to *b* if *a* relates to every object *x* of *C* in exactly the same ways as *b* does. From the point of view of the logic of the structure, then, *a* and *b* are absolutely equivalent, they are substitutable.

Indeed in such a case there is no reason not to identify *a* and *b*. Clearly, the relation of structural equivalence among objects of *C* is an equivalence relation, so that *CObj* can be partitioned into classes of structurally equivalent objects. We can then define in an obvious way a reduced category *Csk*, the *skeleton* of *C*, whose objects are those equivalence classes and whose morphisms and their composition operation are exactly the same as in *C*. Here the reduction mapping of morphisms is the trivial identity mapping and the reduction mapping of objects maps an object of *C* to its equivalence class which is an object of *Csk*. If the nodes of *C* are individuals, then the nodes of *Csk* are groups of structurally equivalent individuals, i.e. maximal relationally homogeneous groups. Concrete examples of such reductions will be described later.

In most social networks represented by a category of the type of C_G no two distinct objects are structurally equivalent. Structural equivalence of nodes usually appears only once a reduction of the morphisms of C_G has been accomplished. Here then two mappings, one of morphisms and one of objects, will be combined into a single reduction operation. Such double mappings exemplify the full notion of *functorial reduction*. The formal conditions which such a double mapping must satisfy are the same as conditions 1 to 3 above, except that the mapping of objects must be applied in addition to the mapping of morphisms when passing from *C* to *D*. Condition 2 must here be interpreted in the context of Nadel's argument that relations between groups of individuals in social systems are based on invariant aspects of, ultimately, relations between individuals (Nadel 1957, pp. 13 f.)

A functorial reduction is a particular kind of *functor*, the only difference being that in a functor neither the mapping of objects nor that of morphisms is required to be *onto*. Thus a functor is any double mapping of objects and morphisms leaving invariant both the distribution of morphisms among objects and the composition operation. (On functors see Mitchell 1965.) Functors furnish the only rational basis for a meaning-

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ful classification of category-like networks. Higher-order categories, where objects are themselves categories and morphisms are functors, and functors between such higher-order categories play an important role in the theory of the lower-order categories.

As a consequence of such double reductions, structural equivalence of objects will vary according to which particular reduction of morphisms is applied. A wider notion of *homothety* is involved here, that is a notion of similarity of position of individuals in a social network, this similarity being relative to particular abstract 'points of view' (reductions) taken on the structure. For example, homothety could correspond to clustering of individuals into disjunct interest groups, common interest being defined by similarity of position relative to some abstract, analytic perspective on the structure. Many such clusterings into interest groups can cross-cut at the same time. Unanticipated equivalences or solidarities in the overall structure between apparently very remote individuals might become visible as a consequence of a reduction. But of course, in general, structural equivalence does not imply actual or conscious solidarity. Concepts of structural equivalence have been used more or less implicitly in the literature; an instructive case is provided by Rosenthal (1968, p. 258) on the subject of coalitions between political parties.

Care must be taken to distinguish being in the same position in a structure and being in isomorphic positions. For example, in a structure consisting of a simple exchange cycle between social groups, no two groups are in the *same* position (i.e. are structurally equivalent), but on the other hand any two groups are in *isomorphic* positions. This could be described as a distinction between *local* and *global* homogeneity.

One can show that if a category C possesses an identity morphism E, whose graph contains the identity relation I, then two objects a, b of C are structurally equivalent if and only if aEb and bEa. In the case of social networks where each type of relation has a unique converse, all identifications among morphisms occur in dual pairs (e.g. $M \equiv N$ and $M^{-1} \equiv N^{-1}$); thus, if E is an image of I (as it will usually be), we must have $E = E^{-1}$, because $I = I^{-1}$: i.e. here the graph of E is more than just reflexive (it includes I) and transitive ($EE = E$), but it is also symmetric, so that it coincides exactly with the graph of structural equivalence among nodes. We now see the importance of identity morphisms in the computation of reductions. In the language of abstract networks of social positions (Lorrain, in press), an identity morphism corresponds to the theoretical possibility of considering the structure from the point of view of a generic ego, either an individual ego or, if structural equivalence is involved, a collective subgroup ego.

The number of possible reductions of a given C_G can be enormous. Thus far we have merely developed a framework for models of structural equivalence. In the choice of reductions lie the main substantive issues.

APPLICATIONS

Criteria for Reduction

A mixture of two strategies may be used in trying to determine meaningful reductions. They will be described in this section and applied immediately in the following sections.

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One strategy we call *cultural*, which looks for possible identifications of morphisms solely on the basis of the cultural content of the morphisms, without regard to their actual graphs. If, in a network of friendship relations, the content of the generic friendship tie F is such that friends of friends are most probably also friends, then the identification $FF \equiv F$ could be made so that in the reduction the F relation would become fully transitive. Or if the generic friendship tie is such that it is symmetric and relatively strong, then the identification $F \equiv I \equiv F^{-1}$ (which also implies $FF \equiv F$ since $II = I$) would be reasonable and in the reduction the classes of structurally equivalent individuals would be cliques of mutual friends. If an 'enemy' relationship N is also given and it is felt that enemies of friends are just as fully enemies as are friends of enemies then the following equations should be imposed: $FN \equiv NF \equiv N$; thus, in particular, composition of F and N would become commutative. It is precisely such a cultural strategy that is used by Lounsbury (1964 a, b) in his analysis of kinship nomenclatures by reduction rules—although there we do not necessarily have a functorial reduction in the strict sense, because the rules are usually ordered according to precedence. One possible starting point for the cultural strategy can be to consider the equations that reduce the infinite set of 'words' to the finite S_G, as done above in deriving S_G for the generators of Figure 1.

The other strategy we call *sociometric*, which considers the actual graphs of the morphisms, identifying those that have many ordered pairs in common, or identifying to I some morphisms that have many self-loops and many symmetric ties in their graph: then in the reduction the nodes linked by these morphisms will become structurally equivalent and hence will be identified. Two morphisms whose graphs have a majority of ordered pairs in common are in similar positions within the composition operation and it is reasonable to identify them in a reduction—the more so as we have already argued that two types of relation with the same graph should be considered a single type.

Note that in a social network represented as a category the graph of a morphism RR^{-1} necessarily consists only of loops and symmetric ties, because $(RR^{-1})^{-1} = (R^{-1})^{-1}R^{-1} = RR^{-1}$. Often such morphisms will be identified to the identity morphism I; we shall do so in an example about to be described. As made clear by Figure 3, $RR^{-1} \equiv I$ implies that any two nodes each related by R to a third node will be structurally equivalent in the reduction and hence identified.

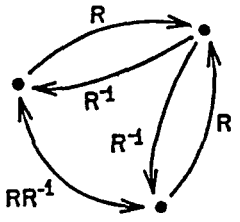

FIGURE 3.

There may be other, better strategies for reduction. Although the two ones just described considerably restrict the number of possible reductions of a network, much remains to be done to relate them more effectively to the actual dynamics of social systems. But, whatever results of this, one thing should by now be obvious:

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identification of morphisms and homothety of individuals are necessarily closely interdependent.

Exploration of possible reductions of a given network is feasible only on a computer. We are currently using programs developed by Heil (1970), which compute reductions of morphisms, structural equivalence, and accomplish various other tasks useful for such an exploration.

Reduction of a Hierarchical Tree

Consider the tree of Figure 1. Let us start from the category C_G generated by P and its converse P^{-1}, computed previously. $P^{-1}P$ is the morphism whose graph (see Figure 2) contains the greatest number of loops and of symmetric ties. Applying the sociometric strategy, let us then pose $P^{-1}P \equiv I$. This single identification reduces the number of morphisms of C_G by more than half; more precisely, all the morphisms in a line of Table 1 become identified. Take, for example, line 4: $P^{-1}P \equiv I$ implies that $P^{-1}P^2 = (P^{-1}P)P \equiv IP = P$ and $PP^{-2}P^2 = PP^{-1}(P^{-1}P)P \equiv PP^{-1}IP = PP^{-1}P = P$. The composition operation of the reduced morphisms is given in Table 4; their graphs are given in Figure 4, where structurally equivalent nodes have been lumped together. Individuals at the same *level* in the hierarchy become identified.

Note that the composition of the *graphs* of two morphisms M, N of this reduction is in general not equal to the graph of the compound morphism MN. No more is required than that the former be a subgraph of the latter. For example the composition of the graph of P^{-1} and the graph of P (see Figure 4) consists of two loops (a, a) and (b, b); however the graph of $P^{-1}P = I$ contains in addition the (1, 1) loop.

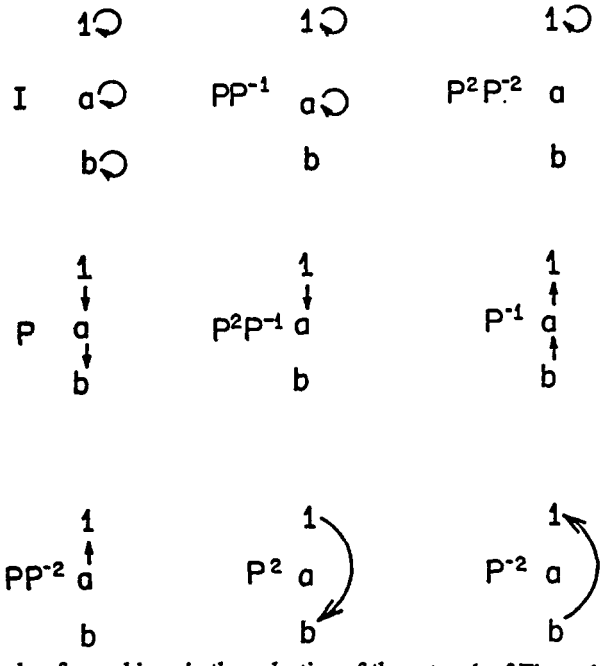

FIGURE 4 Graphs of morphisms in the reduction of the network of Figure 1, implied by the identification $P^{-1}P \equiv I$. a represents the set of nodes 2 and 5, b the set of nodes 3 and 4.

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TABLE 4

Composition table of the functional reduction of the network of Figure 1, implied by the identification $P^{-1}P \equiv I$.

	I	PP^{-1}	P^2P^{-2}	P	P^2P^{-1}	P^{-1}	PP^{-2}	P^2	P^{-2}
I	I	PP^{-1}	P^2P^{-2}	P	P^2P^{-1}	P^{-1}	PP^{-2}	P^2	P^{-2}
PP^{-1}	PP^{-1}	PP^{-1}	P^2P^{-2}	P	P^2P^{-1}	PP^{-2}	PP^{-2}	P^2	
P^2P^{-2}	P^2P^{-2}	P^2P^{-2}	P^2P^{-2}	P^2P^{-1}	P^2P^{-1}			P^2	
P	P	P^2P^{-1}		P^2		PP^{-1}	P^2P^{-2}		PP^{-2}
P^2P^{-1}	P^2P^{-1}	P^2P^{-1}		P^2		P^2P^{-2}	P^2P^{-2}		
P^{-1}	P^{-1}	P^{-1}	PP^{-2}	I	PP^{-1}	P^{-2}	P^{-2}	P	
PP^{-2}	PP^{-2}	PP^{-2}	PP^{-2}	PP^{-1}	PP^{-1}			P	
P^2	P^2					P^2P^{-1}			P^2P^{-2}
P^{-2}	P^{-2}	P^{-2}	P^{-2}	P^{-1}	P^{-1}			I	

Note.—The meaning of the double lines is explained further in the text.

This first reduction of C_G is still somewhat complex for a population of five persons: either they or observers might find the structure difficult to grasp. Now, by definition of a hierarchical tree,† where each person (except the topmost one) has a unique immediate superior, the morphism PP^{-1} occurs exclusively as an endomorphism; from a cultural standpoint, it is then reasonable to pose a further identification: $PP^{-1} \equiv I$. This second reduction, where $PP^{-1} \equiv P^{-1}P \equiv I$, may be considered either as a direct reduction of C_G or as a reduction of the previous reduction where only $P^{-1}P \equiv I$. This is more conveniently accomplished by working with Table 4. In such a table one can verify in the following way if a given partition of the set of morphisms represents a possible reduction.

Consider one such partition, indicated by double lines in Table 4. By extending the double lines of the partition throughout the table supercells are determined. The

†By this we do not mean a transitive relation; in the language of partial orders the tree here is the Hasse diagram of the general authority relation. See Szász 1963, pp. 17 f.

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partition will represent a possible reduction—i.e. one satisfying condition 3 above—if and only if, given any supercell, the morphisms appearing in it are all in the same class of the given partition. Such is the case of the partition of Table 4. But if it had not been the case, for example if our partition had involved only the identification $PP^{-1} \equiv I$, then we should have arrived exactly to the partition of Table 4 simply by grouping certain lines together (and also the corresponding columns together) so that the supercells would satisfy the required property; of course only the groupings necessitated by the original identification must be effected and no others. Thus the double lines in Table 4 give exactly the reduction of C_G implied by the equation $PP^{-1} \equiv P^{-1}P \equiv I$; this is the reduction indicated by brackets in Table 1. This reduction could also have been computed by algebraic manipulation of equations, as in the case of the previous reduction.

Table 5 gives the composition operation of this reduction of Table 4. The graphs of morphisms in this further reduction of C_G are given in Figure 5. These are exactly

TABLE 5

Composition table of the functorial reduction of the network of Figure 1, implied by the identifications $PP^{-1} \equiv P^{-1}P \equiv I$.

	I	P	P^{-1}	P^2	P^{-2}
I	I	P	P^{-1}	P^2	P^{-2}
P	P	P^2	I		P^{-1}
P^{-1}	P^{-1}	I	P^{-2}	P	
P^2	P^2		P		I
P^{-2}	P^{-2}	P^{-1}		I	

the global roles which one would expect, in this case. Later on we shall effect even further reductions of this reduced category.

Similar results are obtained whenever the identifications $PP^{-1} \equiv P^{-1}P \equiv I$ are made in a tree (see Footnote p. 67). All individuals at the same level in the tree (counting levels from the top) become structurally equivalent and are identified: this is because $P^{-1}P \equiv I$ implies that, for all positive integers k, $P^{-k}P^k \equiv I$. If there are $n+1$ levels, then there are exactly $2n+1$ morphisms: $I, P, P^2, \ldots, P^n, P^{-1}, P^{-2}, \ldots, P^{-n}$. The composition operation is simply the following: given integers i, j (positive or negative), P^iP^j, if defined, is equal to P^{i+j} (here we pose $P^0 = I$). Composition thus is commutative.

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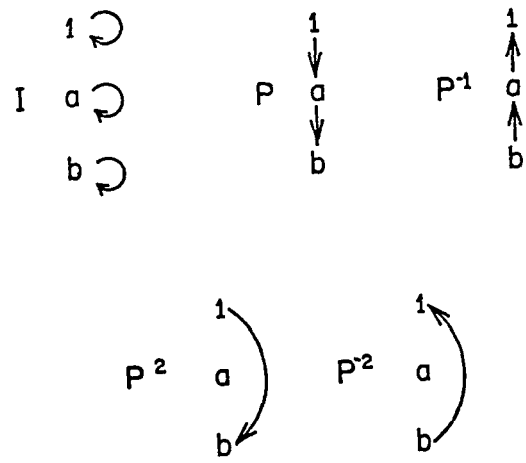

FIGURE 5 Graphs of morphisms in the reduction of the network of Figure 1, implied by the identifications $PP^{-1} \equiv P^{-1}P \equiv I$. a and b as in Figure 4.

Note that if we added to these morphisms a zero and defined $MN = O$ whenever the compound MN is undefined, then the result would not be a semigroup, if $n > 0$, because then composition would not be associative: e.g. $P^n(P^nP^{-n}) = P^nI = P^n$ but $(P^nP^n)P^{-n} = OP^{-n} = O$. Note also that this does not contradict point 3 of the definition of a category.

Such a reduction of a tree is very meaningful in organizations such as armed forces, where every person at a given level is expected to obey the orders of any person at a higher level. Moreover, in such a case, the reduction constitutes a time-invariant picture of the hierarchy, which remains the same whatever be the changes in number of branches, span of control, etc., so long as the number of levels is not changed.

Kinship, Role Trees

Certain kinship systems are based on identifications of the type $RR^{-1} \equiv I$. Denote by C the father-child relationship and by W the husband-wife relationship. Applying a cultural argument, we can impose $CC^{-1} \equiv I$. *Classificatory* kinship systems are those where in addition $C^{-1}C \equiv I$ is required, so that siblings are structurally equivalent and groups of siblings are considered as units in the system; this is Radcliffe-Brown's principle of the unity of the sibling group (Radcliffe-Brown 1950, pp. 23 ff.). A refinement of this framework is possible where siblings are distinguished by sex. If in addition $WW^{-1} \equiv W^{-1}W \equiv I$ is required, then the society is partitioned into 'marriage classes': there is a unique predetermined class from which a woman of a given class must take her husband, and conversely. This would be a society with strict prescribed marriage. Such structures—call them *elementary* kinship systems—have been extensively studied by Lévi-Strauss (1949) and by White (1963); Lévi-Strauss' definition of an elementary kinship structure is less restrictive, however.

Other identifications occur frequently in elementary kinship systems. One example is $WC \equiv CW$, corresponding to a rule of matrilateral cross-cousin marriage. Another

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example is $C^n \equiv I$, for some positive integer n; this means that successive generations in a given patrilineage will be arranged along a cycle of length n, the $(n+1)$-th generation being in the same kinship class as the first, etc. The case $n = 2$ is particularly frequent and is referred to in the literature as a system with 'alternating generations'.

When such 'cultural' identifications are applied to an infinite abstract role tree containing all possible kinship roles relative to a generic ego, they amount to folding in and rolling up the tree in a systematic way such that in the end a finite structure is obtained. The intuitive verbal analysis of White (1963, Chapter 1) amounts precisely to this and such an operation can be described as a functorial reduction (see Lorrain 1970).

Such a finite elementary kinship structure can also be obtained 'sociometrically' as a functorial reduction of the network of kinship relations in the concrete population, as shown by Boyd (1969) in the language of semigroups and homomorphisms.† The elementary kinship structure is then seen to represent a time-invariant reduction of the detailed kinship network in the population.

Classificatory Reduction

Reduction of a tree to a linear ordering of levels, considered previously, is a prototype of a more general kind of reduction, *classificatory reduction*, the result of which is a type of category called a *quasi-homogeneous space*, of which elementary kinship structures, componential analyses, and classical affine space are examples. The classificatory reduction of a category that satisfies the 'principle of reciprocity' results from identifying *all* endomorphisms to the identity; this represents an extreme form of the sociometric strategy. It is possible (see Lorrain, in press) to construct, using this notion of classificatory reduction, a theory embracing the core of classificatory kinship nomenclatures, elementary kinship structures, systems of binary oppositions, componential analysis, hierarchies with levels, and balance theory. Such a theory offers multiple points of view on the same structures and can be expressed in dual ways, emphasizing either relations distributed among pairs of individuals or individuals allocated to social positions.

Restrictions on the Composition Operation

Only in kinship networks or in formal hierarchies are very long 'words' known to represent a meaningful type of relation. This suggests limiting the length of words, by considering a long word to be 'undefined' whenever its graph is distinct from the graph of any shorter word. This would give rise to a truncated composition table, in which reduction of morphisms could proceed as usual. A word is perhaps not very significant if it has more letters than there are nodes in a given network; word length should be limited still more drastically if attenuation of indirect structural effects with length of path is considered severe. The model then would focus more on local structure, overlap between successive neighborhoods being the basis of any long range order; in extreme form the model collapses back to a more conventional sociometric

†There is however an error in Boyd's theorem (1969, p. 145), which was not in a previous mimeographed version of that paper: a condition must be added, so that the operators considered in the theorem are everywhere defined.

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analysis of graphs. For example, step by step transmission of social influences—say diffusion or gossip—is probably adequately captured by looking at generator ties alone, so long as there are no regular patterns of alliance and compartmentalization. But it is exactly such patterns which can be found by the search for models of structural equivalence from the category C_G.

In some situations it might prove necessary to define a new type of binary relation reporting whether an individual knows another well quite aside from the feelings and expectations they have for one another. Then only those generator ties that coincide with knowledge ties would be permitted as first steps in compound ties. Such a principle has been applied in a sociometric study by White (1961). It might even prove necessary, in some cases, to include data on perceptions of ties by third parties to obtain more realistic accounts of which secondary ties exist and are effective.

Reduction of an Actual Social Network

The network considered here is abstracted from data collected by Sampson (in press).† The population consists of the members of a monastery. This group was a residual of a long period of turmoil and conflict between a more progressive tendency —the 'young turks'—and a more conservative one—the 'loyal opposition'. It continued as a group for a relatively short time (approximately two weeks), during which it remained isolated from previous members and after which is disintegrated. However, although there was not time enough for social integration to come to conclusion, the relations measured were considered clear enough to warrant an analysis by categorical-functorial methods. Two generator relations are considered, P and N, whose graphs are given in Figure 6. *aPb* means that *a* likes *b* most, is influenced by him most, or sanctions him positively most frequently; P is thus a general positively oriented relationship. *aNb* means that *a* likes *b* least, esteems him least, or sanctions him negatively most frequently; N is thus a general negatively oriented relationship.

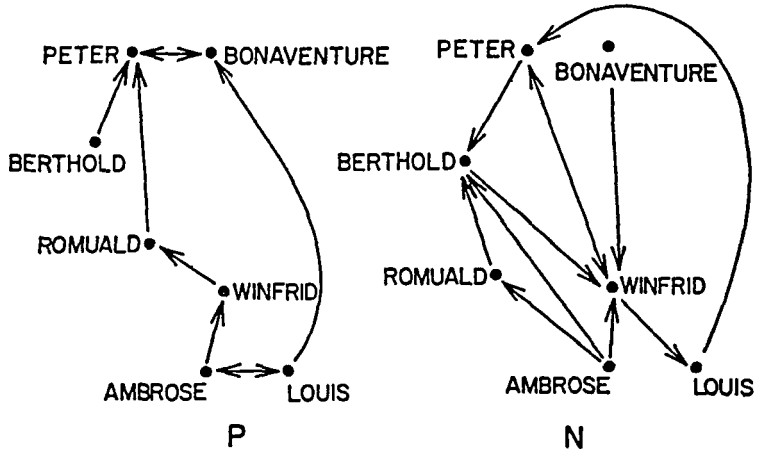

FIGURE 6 Positive and negative relations in a monastery.

†Interpretation of the results was greatly facilitated by the kind help of Dr. Sampson.

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Other degrees of the relationships were in the data, but they were excluded for the purpose of the present analysis. Moreover, because these two relations do not correspond to a well enough delineated set of social expectations, their converses were not included as generators.

In spite of there being only two generators, the number of morphisms in C_G is enormous, probably at least several hundred. Accordingly, the length of 'words' was limited to between five and six letters, keeping only seventy morphisms. But the reductions that were then operated were so drastic that this limitation did not change the final result to the slightest extent.

Reductions of C_G to categories with four or five morphisms only were first effected, by applying the sociometric strategy. For example, since P and P^3 have eight ties in common as compared to twelve ties in one or the other, P was identified to P^3. This implies $P^2 \equiv P^4$, but these two morphisms have exactly the same graph, so that they are already considered equal in C_G. N^2, N^3, N^4, and N^5 have between 60 and 97% of their ties in common, so that the identification $N^2 \equiv N^3$, which implies $N^3 \equiv N^4 \equiv N^5$, was imposed. And so on. However these reductions with four or five morphisms were still difficult to interpret and further reductions were operated. Only four non-trivial ones were possible, two with three morphisms and two with two morphisms. The latter two are given in Figures 7 and 8, where the same labels P and N are retained for their images. In those reductions structural equivalence does not depend on an identity morphism, none having been included in C_G.

In the reduction of Figure 7, N has become a universal neutral relation linking every node to every other node. This neutrality is obvious in the composition operation

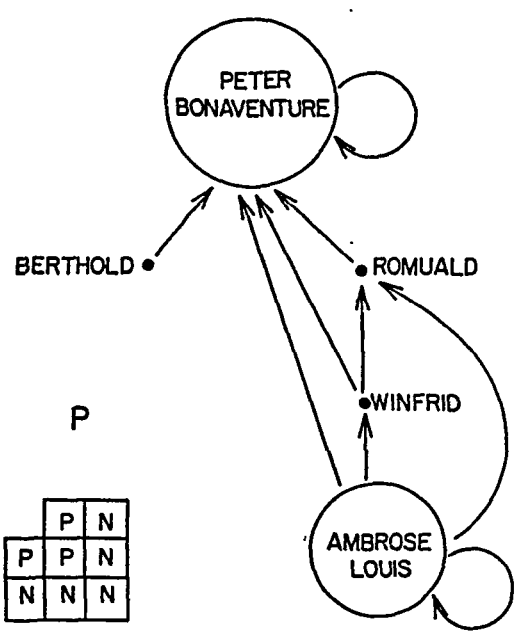

FIGURE 7 A first example of reduction of the network of Figure 6. Only the graph of P is shown; the graph of N is a trivial one linking every node to every other node.

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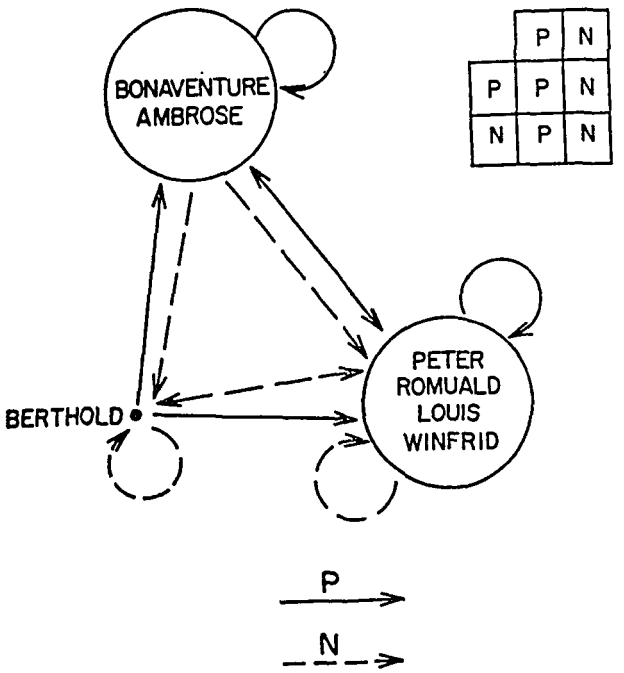

FIGURE 8. A second example of reduction of the network of Figure 6.

of this reduction, where N acts as a zero morphism. P is transitive, so that the structure takes the form of a partial order. It is significant that Peter and Bonaventure become identified: they were clearly the leading, most influential elements of the group at that time, although for very different reasons, Bonaventure being rather above the battle and Peter leading the 'loyal opposition'; note that Peter and Bonaventure are not equivalent in the reduction of Figure 8.

In relation to Figure 7, it is remarkable that Romuald and Winfrid were the first to leave the group. Romuald had come late and was caught between the two tendencies and Winfrid had been very much associated with the 'young turks'. On the other hand Berthold, a member of the 'loyal opposition', was really a special case, close to Peter, but very isolated from the others; Ambrose, although linked positively to one of the 'young turks', was a member of the 'loyal opposition', closely tied (symmetrically) to Louis, who was closest to Bonaventure. As we shall see in the next section, the intermediary nodes in an ordering (such as in Figure 7) are really in quite a particular position: structurally speaking, and however paradoxical this may seem, the two extremes are in fact closer to each other than to the intermediaries and, in so far as the structure is correctly represented, they are more likely to ally, leaving the intermediaries to themselves. Such a situation was certainly the case in the group considered and seems to be reflected in the structure of the reduction of Figure 7. As a matter of information, Peter was the third person to leave, having obviously lost his function as leader of the opposition as soon as Romuald and Winfrid were gone.

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The reduction of Figure 8 is quite interesting; in particular the clusters of nodes are totally different from those in the reduction of Figure 7. Unfortunately our lack of familiarity with the concrete social situation of which these networks are abstractions prevents us from a full interpretation. Nevertheless, it is remarkable that Berthold once again appears as an isolate and that Romuald and Winfrid, those closest to the 'young turk' tendency and who were the first to leave and also Peter, whose departure followed immediately, are grouped together. The case of Louis being also lumped with these three is rather puzzling, however. Note also the 'ambiguous' (*P and N*) relations linking both the two-men node and the one-man node to the four-men node and also linking the latter to itself. Of course P and N must be understood as 'positive' and 'negative' only in a very generalized sense; their reality could hardly be assessed by the more impressionistic methods of observation. Note also that the graphs of P and N in this reduction are isomorphic although one appears rotated relative to the other.

The composition operation in Figure 8 is also interesting, especially if we consider its representation as a graph in Figure 9. This graph represents the 'role' system involved in the social network of Figure 8; roles are themselves related by role relations, because of the interlocks represented by composition: e.g. N links P to N, in Figure 9, because $PN = N$ (see Figure 10 for a more inspiring visual representation of a role relation linking two roles). Figure 9 shows a 'positive' and a 'negative' role: the positive one relates positively to itself and negatively to the negative one, the negative role relates positively to the positive one and negatively to itself. This is a rather familiar type of social situation.

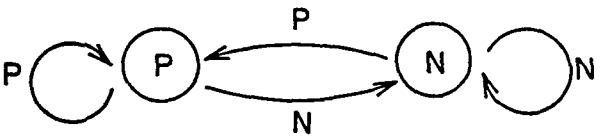

FIGURE 9 Graphic representation of the composition operation in Figure 8.

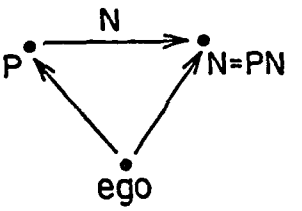

FIGURE 10 Roles related by a role relation.

On the Solidarity and Alliance of Extremes

Consider the linear hierarchy category of Figure 5 and Table 5; denote it by H_0. The sole identification $P^2 \equiv P^{-2}$, without any other identifications, defines a possible reduction of H_0, as is easily verified by glancing at Table 5. One can also see from Table 5 that this reduction—denote it H_1—is the one nearest to H_0, in the sense that any other reduction of H_0 involves more than one identification.

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The composition operation of H_1 is given in Table 6. Note that in H_1 the compound of any two morphisms is defined, i.e. any 'role' can interlock with any other 'role'. Moreover, as in H_0, composition in H_1 is commutative. P^{-1} can now also be written as P^3. This composition operation is isomorphic to that of the cyclic group of order four, i.e. to the table of addition of integers modulo four: simply write 0 instead of I, 1 or -3 instead of P, 2 or -2 instead of Q, 3 or -1 instead of P^{-1}. No further structural equivalence of objects appears when passing from H_0 to H_1.

TABLE 6

Composition table resulting from identification of P^2 to P^{-2} in Table 5.

	I	P	Q	P^{-1}
I	I	P	Q	P^{-1}
P	P	Q	P^{-1}	I
Q	Q	P^{-1}	I	P
P^{-1}	P^{-1}	I	P	Q

Note.—Q is the common image of P^2 and P^{-2}.

A symmetric relation such as Q between the two extremes of a hierarchy is not a rare occurrence in classificatory kinship systems and is referred to as 'intergenerational solidarity'. There is an obvious analogy to the ideology according to which citizens can be called the masters of their top executives, the servant of the people.

Now only one non-trivial reduction of H_1 is possible: it is the one where $I \equiv Q$ and $P \equiv P^{-1}$. Call it H_2. H_2 has two morphisms, I and R, with I and Q of H_1 mapping to I of H_2, P and P^{-1} of H_1 mapping to R of H_2. The composition operation is $II = RR = I$, $IR = RI = R$. This is isomorphic to the composition operation of the cyclic group of order two, i.e. to the addition operation of integers modulo two. In H_2 nodes 1 and b are structurally equivalent so that H_2 takes the form of two symmetrically related moieties (see Figure 11). If R were a 'negative' relation and I a 'positive' one, then this would be a classical situation of 'structural balance', a notion examined in the next section. If H_0 represented a segment of a kinship lineage, H_2 would correspond to a case of 'alternating generations', mentioned earlier.

We now clearly see in what sense we could say, in the last section, that the extreme positions in an ordering are structurally closer and the intermediary ones less 'stable', perhaps more likely to become opposed to both extremes in the case of conflict. Of course this makes sense only in so far as the given structure is correctly described by such a hierarchical arrangement.

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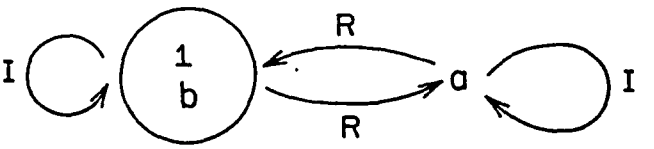
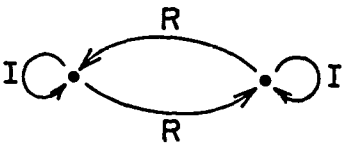


	I	R
I	I	R
R	R	I

FIGURE 11 A reduction of the category of Figure 5 and Table 5.

Balance Theory as a Special Case of Functorial Reduction

At first sight balance theory is a simple variation of graph theory and thus has little connection with the elaborate algebraic structures derived from role graphs. Further inspection shows that balance theory, in the formulation of Abelson and Rosenberg (1958), can be expressed as a special kind of functorial mapping of a category. The image category is required to be just the graph of the cyclic group of order two (see Figure 12)—call it the *two-category*. Only two generator roles are allowed, both symmetric so that each is its own converse. Further, one particular generator role, that thought to represent positive feeling, is mapped to the image identity. The other generator is mapped to the other morphism in the two-category. Compound roles are not defined explicitly, but all ties corresponding to paths with the same structure are mapped to the same image element. The 'addition' of different ties between a given pair corresponds to seeing if they all map to the same image element. The requirement that all closed cycles be 'positive', in Cartwright and Harary's (1956) graph-theoretic formulation, corresponds to an extreme form of the 'sociometric' criterion for a reduction used earlier: this requirement means that all endomorphisms



	I	R
I	I	R
R	R	I

FIGURE 12 The two-category: the graph of the cyclic group of order two. This category is isomorphic to the one in Figure 11.

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are mapped to the identity; this is a case of 'classificatory reduction'. Thus we see how such a functor partitions the set of nodes into two classes, positive relations holding only within the same class and negative relations linking only individuals in distinct classes.

Balance theory shares the weaknesses of the categorical-functorial approach (see discussion in conclusion) but has a number of further weaknesses. If a structure is not 'balanced' resort must be had to measures of the 'amount' of imbalance, and adequate measures of this type remain to be devised (see Boorman 1970). Whereas in the categorical approach one can look for the coarsest possible non-trivial functorial reduction and the resulting model of structural equivalence can just as reasonably be called 'balanced', whatever the number of equivalence sets and image relations.† In the categorical approach there is no need to require any of the generator roles to map to the image identity. For many theoretical purposes it is more fruitful to require not a direct tie but having direct ties to the same person as the criterion for being together in a clique. Little systematic work on extending balance theory to non-symmetric roles has been published, and never more than two generator roles are used.

Nothing is lost by using the categorical approach. Search for a reduction to the two-category will show whether the structure is in fact 'balanced' and, if so, produce the required two cliques. The reduction of Figure 11 is an example; the cliques here are 1, 3, and 4 versus 2 and 5 (see Figure 1). Even in this simple example something is gained by the categorical approach. The cliques and the two image roles are derived from data on role ties that need not be symmetric nor have any positive or negative affective qualities. Instead one can discern on structural grounds a likely line of cleavage which may become realized in explicit social relation only later.

The generalization of balance theory proposed by Davis (1967), which includes the possibility of there being more than two cliques, can also be represented by functors. However, here not all endomorphisms will be mapped to the identity.

CONCLUSION

Networks will probably become as important to sociology as Euclidian space and its generalizations are to physics. Unfortunately the mathematical theory of networks is far from having attained a degree of development even remotely comparable to that of modern topology and analysis. Graph theory (Berge 1967, Ore 1962) has little unity and has little to say on interrelations between relations, dealing essentially with isolated graphs; moreover it does not offer any criteria for reduction and comparison of graphs. Little motivation seems to have developed within the sociometric tradition to study interrelations among relations. However, papers by Davis and Leinhardt (see Footnote p. 52), Holland and Leinhardt (see Footnote p. 51), and Boyle (1969) are stimulating exceptions to this.

The categorical-functorial framework, at least as applied here, has several weaknesses. One is that it seems insensible to distance between nodes in the network of

†A manuscript by J. A. Davis entitled 'Boundary Relationships' (1965) develops similar ideas in terms of multiple graphs.

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generator ties, although of course the unreduced C_G gives full account of such distance. Certain network properties seem not to be characterizable by the composition operation of morphisms. Friedell (1967) has suggested that semilattices, a type of partial order less restrictive than a tree, may be a plausible representation of the reality of authority structures when individuals have more than one immediate supervisor; however, it is impossible to characterize a semilattice only be the composition operation of its representation as a category (Lorrain 1970).

Strength of ties and intensity of flows should really be handled quantitatively. It should be feasible to treat in a systematic and global manner networks where numbers are associated to ties. Representation of these networks as tensors and the use of tensor fields is one possible formalization.

Although individual attributes such as sex can easily be represented by differentiating ties according to the sex of their endpoints, this makes less sense for more continuous attributes such as age. However around visible attributes such as age, race, or sex there often cristallize crucial ideologies of role differentiation, of inferiorization, or of exploitation. There seems to be no obvious general way to integrate attributes to a network representation. This is a real and difficult problem, even though we have argued that social differentiation is essentially a function of the interweaving of social relationships.

Another important direction for development of functorial reduction ideas would lie in formalization of numerical reduction criteria, e.g. of the idea of the 'simplest' reduction within a certain structural 'distance' of an initial category. A collection of models of structural distance is analyzed in an integrated fashion by Boorman (1970), who shows how simple metrics on sets and partitions naturally generalize to highly interpretable and computable metrics on semigroups and related algebraic objects. At the same time, developments of this approach could provide much needed stability for categorical network theory. Even a single exception to a rule generally valid in a social system represented as a category can totally disrupt its reduction, even making it trivial: it should be possible to approach the problem of 'exceptions', as well as the problem of measurement error, with the help of appropriate measures of structural approximation. These metric concepts could also be used to evaluate relative degrees of structural equivalence among the elements of a structure.

In a sense, our approach requires homogeneity of point of view among the members of a social network. However this really applies only to generators and, even if all individuals agree in their perception of generator ties, competing ideologies differing in their global interpretation of the system are still conceivable. A social network does not form a unitary block. A network in fact *consists* of holes, decouplings, dissociations; ties can reflect conflict as well as solidarity, they reflect interdependence, not necessarily integration. Numerous points of view on a network are possible, latent lines of scission can be drawn. In short, our notion of network is closest to a dialectical notion of totality (see Lefebvre 1955), and as such could provide the foundation for a treatment of social dynamics. As has been shown in the case of language (see Jakobson 1957, p. 10), synchrony does not coincide at all with statics, in systems where there is relative decoupling or autonomy of parts: conceptions of synchronic structure are of crucial importance in the question of dynamics, and conversely.

Nevertheless, in this paper we have not really faced the question of diachrony. To

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do so it would be particularly important to consider the responses of structures to—and their shaping by—such driving forces as the demographic factors of birth and death, the inputs from concrete everyday social practice, and the absolutely crucial factor of material resources. The dual processes of allocation of individuals to social positions and of social positions to individuals would be a central concern here, special attention being paid to actual numbers—such is the subject, for example, of White's *Chains of Opportunity* (1970). Links should be made explicit between these considerations and mathematical theories of *morphogenesis* in dynamical systems (Thom 1968, 1969).

The main strength of the categorical-functorial approach is in locating sets of individuals, however large or small be the direct distance between any two in a set, who are placed similarly with respect to all other sets of individuals, to the extent that total relations and flows are captured by the aggregation of detailed relations consonant with those equivalence sets of individuals. The notions of class society and of imperialism are prototypical examples of such global structural representations. Structurally coherent solutions encompassing all individuals at once, rather than successive calculations of connections between more and more remote individuals, are the goal of the analysis.

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